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Challenges and Opportunities for Existential Graphs

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1. Introduction

In 1896–1911, Peirce developed a number of novel systems of diagrammatic logic, commonly known as Existential Graphs (EGs). He divided them to the alpha, beta, gamma and delta parts. They contain, among others, the diagrammatic counterparts to propositional logics (alpha), fragments of first-order logics with identity (beta), modal and quantified multi-modal logics, higher-order logics, meta-assertions similar to Gödel numbering, and logics for non-declarative assertions (gamma/delta).

Despite obviously being ahead of their time, EGs have played an unusual role in the early development of modern symbolic logic. I will deal with this curious history in a sequel to the present paper. I will confine the present study in what I take to be the major challenges as well as opportunities this recently resurrected diagrammatic and iconic logical method faces from the points of view of contemporary philosophy of logic, reasoning and cognitive representation.

Hammer's (2002) review of some basics of EGs ends with a dissuading note: "a diagrammatic logic is simply a logic whose target objects are diagrams rather than sentences. Other than this, diagrammatic logics and logics involving expressions of some language are not different in kind" (Hammer, 2002, p. 421). The purpose of the present paper is also to demonstrate that the relationship between diagrammatic and sentential approaches to logic is not at all as straightforward and simple minded as Hammer would have us believe.

I will assume that the basics of the theory of EGs are known to the reader (see, e.g., the bibliography of Liu, 2008).

2. The iconicity of logical constants

Peirce desired EGs to function as a new way of expressing logical notions in a diagrammatic, spatial, topological and iconic instead of the unilinear and symbolic manner. However, he did not come to contemplate them merely to create an alternative notation by means of which to do logical modeling, linguistic representation or deductive reasoning. He was simply not well versed to grasp the meaning of linguistic expressions if one has to stick to the symbolic and serial modes of expressions:

> I do not think I ever *reflect* in words: I employ visual diagrams, firstly because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose. MS 619:8 [1909], "Studies in Meaning"

Earlier, he had defined such schematizations to be diagrams that are certain iconic representations of facts and which may, but need not be, visual:

> We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination.
> CP 2.778 [1901], "Notes on Ampliative Reasoning"

It was essential towards realizing his goals that all logical notions and conventions are given a solid philosophical justification. In 1902, Peirce published the article "*Symbolic Logic* or *Algebra of Logic*", co-authored with his former student Christine Ladd-Franklin, in the influential and widely referenced Baldwin's *Dictionary of Philosophy and Psychology* (Peirce 1902, pp. 640-651, printed, with omissions, in CP 4.372-4.393). That article presents a comprehensive exposition of the propositional (alpha) and the first-order (beta) parts, including a complete proof system for the alpha graphs, and not only. Diagram logics are subsumed under the wider notion of an "analytical" method for representing logical ideas, the purpose of which is "simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences" (Peirce, 1902, p. 645). The article recognizes it "as a defect of a system intended for logical study that it has two ways of expressing the same fact" (ibid., p. 645), whereas diagrammatic notions can unify what under the calculus conception would involve different logical constants for the expression of the same facts.

For example, the soon-to-be-emerging design of formalized conception of logic has the defect of ripping apart one underlying fact of the logical universe of discourse, turning the parts into the separate notations for existence (existential quantification), predication (predicate terms and bound variables) and identity (a special two-place relation). In EGs these are all expressed by the same, iconic sign of the line of identity. The result is said to be "by far the best *general* system which has yet been devised" (Peirce, 1902, p. 649) and "the only perfectly analytic method of logical representation known" (MS 284 [1905], "The Basis of Pragmaticism"). Other notions of symbolic logic, which were soon to find their foundational value in being able to ape mathematical calculi, would have to be rated not "as much higher than puerile" (MS 499 [1906], "On the System of Existential Graphs Considered as an Instrument for the Investigation of Logic").[1]

Yet soon after Peirce's death, the focus on logic had already turned to other matters. Fueled by the reception of Wittgenstein's *Tractatus* (1921), Bertrand Russell had launched a new campaign promoting the idea of uninterpreted, purely formal languages. It turned its back on the algebraic tradition, the birthplace of diagrammatic logic, in its redefinition of symbolic logic. Peirce's 1902 entry on symbolic logic in Baldwin's *Dictionary* conjoined "symbolic" with "diagrammatic" and thus with algebraic thinking.[2]

And so symbolic logic came to take a different turn from the prospectus set out in the dictionary article. That article placed a great importance

[1] The allusion is to Peano's pasigraphy: "Peano's system is no calculus; it is nothing but a pasigraphy; and while it is undoubtedly useful, if the user of it exercises a discrete freedom in introducing additional signs, few systems of any kind have been so wildly overrated, as I intend to show when the second volume of Russell and Whitehead's *Principles of Mathematics* appears" (MS 499). For Peirce logic is "not intended for a plaything"; it is neither any universal system of expression nor a calculus in its limited sense: "This system [of logical algebras and graphs] is not intended to serve as a universal language for mathematicians or other reasoners, like that of Peano. [And this] system is not intended as a calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression" (CP 4.424 [c.1903]). To these two requisites Peirce adds that he has excluded any considerations of human psyche that may have been involved in those traits of thinking that led to the inventions of the signs employed in his systems of EGs.

[2] A singular reason for Russell's sea change seems to have been Peirce's dismissive October 1903 book notice in the Nation on his *Principles of Mathematics*, which according to F. C. S. Schiller had driven him "hugely annoyed" at once (Schiller to Welby, 26 November 1903; see Pietarinen 2009).

on the iconic nature of logical thought – not that different from contemporary semantic and model-theoretic perspective (Pietarinen, 2006a) – while the soon-to-be-prevailing Frege–Russell conception was calculated to begin the theory development with uninterpreted constants and rules of inference. Peirce surely recognized the interest in such purely formal rules as such. At one point he termed them the "Code of Archegetic Rules" of transformation (MS 478:151).[3] He then delineated the "purely mathematical definition" of EGs "regardless of their interpretation" (MS 508), which will be useful to portray the proof-theoretic components of the general theories of the alpha, beta and gamma parts.[4] But such an uninterpreted language alone would not meet the ends and purposes of *theorematic* logical reasoning. Likewise, any unrestrained acceptance of uninterpreted non-logical vocabularies to logical studies would have countered his entire project of being able to conceive "logic as the theory of semeiotic", without which genuine scientific discovery would not be possible at all (MS 336 [c.1904], "Logic viewed as Semeiotics"; cf. MS 337).

In sum, EGs imply the failure of what Hintikka (1979) has termed the Frege-Russell ambiguity thesis. The thesis states that the verb for *being* is multiply ambiguous and that the logic should reflect the underlying logical difference between the multiple uses the verb for *being* has. However, in EGs, the line of identity represents predication, identity, existence, and class-inclusion, all in one go. Consequently, it is the one logical sign of a line that is able to capture all the varieties of *being*.[5]

The tendency towards greater unification and simplification in logical notation has not only the benefit of greater cognitive economy and efficiency of expression and communication of diagrammatic assertions but is also something necessitated by the age-old Aristotelian understanding of

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[3] The systems of transformation rules is sound, since "the rules are so constructed that the permissible transformations are all those, and all those only, by which it is logically impossible to pass from a true graph to a false one." This explanation "is no part of the rules, which simply permit, but do not say why" (MS 478:150). The system of rules is announced to be complete by virtue of the fact that "none of its rules follows as a consequence from the rest, while all other permissibilities are consequences of its rules" (MS 478:151).

[4] That part of the gamma part I have in mind here concerns the "potentials" and not the broken-cut modal logics, the former of which give rise to higher-order graphs in which the quantificational lines refer not to individuals but to what according to Peirce's curious remarks are the "strange kinds" of "proper names" that are "substantive possibilities" and devoid of individualities (MS 508, "Syllabus B.6").

[5] We can add anaphora here, too: "A dean dances in the park. He sings" = "A dean dances in the park and is singing". So aside from predication, identity, and existence, the same notation takes care of coreference, too.

being qua being, represented as lines qua lines and not through anything else. However, the fact that there are multiple yet logically equivalent readings of graphs does not imply ambiguity in such representations. Unlike Shin (2002), we need not puzzle over what the "visually clear" and "intuitive" ways of "reading off" these graphs may be. Ambiguity is a natural-language phenomenon and as such does not carry over to the iconic realm of diagrammatic expressions.

My argument above is thus merely to accentuate the importance of understanding the meaning of EGs as icons, not through translation to sentences of propositional, first-order or modal logics. Diagrams appear to enjoy such cognitive economy that is hardly encountered in the convention-based symbolic systems of logical languages.

What, then, is the meaning of logical constants? Where does their meaning come from? A new answer could be sought for in the diagrammatic iconicity of logical expressions. Negation, conjunction, implication and quantification are iconic signs and hence capable of expressing their own meaning. Negation is an operation of incision of an area of a graph from the space of assertion in question followed by a reversal of that area; conjunction is juxtaposition of assertions in the space; given two nested cuts, implication is ability to continue a passage from the area of an outer cut area to the area of the inner cut; quantification is a dot or a continuous line the extremities of which hit upon certain elements in the domain of discourse of the topological manifold of all potential assertions. Hence the meaning of logical constants is not something that follows from inference or transformation rules.

A further argument supporting my result is that it is impossible to diagrammatize the infamous TONK connective by any transformation rules. That connective takes the introduction side from the disjunction rule and the elimination side from the conjunction rule and merges these into the one mock rule for the TONK connective. However, there is no way of erasing a negated graph from a negative area which is not a result of any iteration.[6] As far as the meaning of logical constants is concerned, therefore, the iconicity of logical signs in EGs makes use of the nature of the space within which they are scribed and hence is a feature that has to precede any conception of the deductive component of transformations.

The fact that logical constants may be spatial arrangements that need not follow the linearity of time was much later reaffirmed by Enderton:

[6] At least not in two or three-dimensional, instead of four or higher-dimensional sheets of assertions.

> We speak in real time, and real time progresses linearly. [...] But formal languages are not spoken (at least not easily). So there is no reason to be influenced by the linearity of time into being narrow-minded about formulas. And linearity is the ultimate in narrowness.
>
> Enderton, 1970, p. 393

Enderton refers here to partially ordered quantifiers, which as noted may well provide a symbolic counterpart to those "stereoscopic" graphs Peirce's alluded to in a June 1911 letter (MS L 231). In the letter, he in fact maintains that the rational parts exhibited in diagrammatic syntax "are really related to one another in terms of relations analogous to those of the assertions they represent," and hence, "in studying this syntax we may be assured that we are studying the real relation of the parts of the assertions and reasoning," which is not the case "with the syntax of speech" (MS L 231:10). He notes the syntax of speech to be restrictively linear, much like two-dimensional algebra is in comparison with, say, the topological higher-dimensional algebras.

At present we certainly have the 'heterogeneous' logics at our disposal (Barwise & Etchemendy, 1995; Shin, 2004). But they are not iconic in the full sense of the term. They combine diagrammatic with symbolic signs, and replace some of the constituents, such as predicates, which in EGs are non-diagrammatic icons of images, with symbolic notations. Conversely, symbolic logic is heterogeneous in the sense that for instance algebraic, model-theoretic, and inferential thinking all appeal to diagrammatic conceptualizations. At the same time, EGs do not claim to be completely and purely iconic, either, but to strive to be "as iconic" representations of logical thought "as possible". Iconicity is needed to represent relations by "visible relations analogous" to the intended, actual relations in the model (MS 492:22). Such profound iconicity is further related to the idea of diagram construction and assertions as utterances employing "any method of graphic communication" (MS 492:24). Peirce operationalizes this communicative aspect of the meaning of graphs by imagining a dialogue that takes place between the utterer and the interpreter, an idea taken from his algebra of relatives and explicates in terms of a new 'interactive' semantics for EGs (Peirce, 1906; MS 280; Pietarinen, 2006a).

Here emerges our first challenge, then:

> **Challenge A:** *Tackle the current question of the meaning of logical constants from the point of view of the diagrammatic-iconic method, capable of unifying the signs of logic.*

3. The logic of cognition

Peirce's goal was to develop a comprehensive logic of cognition through iconic means. To accomplish this, the workings of the information processes in cognition need analysis in a rigorous and structure-preserving fashion even when symbolic expressions fall short of fulfilling that purpose. And they shall fall short, Peirce avers, since "there are countless Objects of consciousness that words cannot express; such as the feelings a symphony inspires or that which is in the soul of a furiously angry man in the presence of his enemy" (MS 499).

The possibility of an iconic logic of thought means that the essential representational and inferential aspects of the processes of the mind can be articulated by certain specific kinds of diagrams. According to Peirce, logical diagrams are precise snapshots of thoughts mind produces. On the contents of minds diagrams give "rough and generalized" pictures (CP 4.582), which nevertheless are logically as precise as any conceptual or abstract framework can possibly reveal. The reason is, he explains, that diagrams are icons that reflect continuous connections between "rationally related objects" (MS 293:11). Our knowledge about rational connections comes not from experience or mathematical certainty, but from something "which anybody who reasons at all must have an inward acquaintance with" (MS 293:11; Pietarinen, 2005b).

With EGs, one is equipped to represent and investigate analytically "all that is in any way or in any sense present to the mind" (CP 1.284). Interestingly, EGs live at the core of the principle of pragmaticism and Peirce appealed to them in his attempts to prove that pragmaticism is in fact the true theory of meaning: "The study of that system must reveal whatever common nature is necessarily shared by the significations of all thoughts. [EGs] furnish a test of the truth or falsity of Pragmaticism [by disclosing] what nature is truly common to all significations of concepts" (MS 298 [1905], "Phaneroscopy").[7]

EGs thus provide grounds for Peirce's announcement that they are the real representations of our "moving pictures of thought". But exactly how do they do it? EGs seem to deal with some vital aspects of information flow and information processing. Two aspects are worth highlighting here.

[7] The detailed reconstruction of the intended argument is beyond the scope of the present paper (see Pietarinen, 2010a; Pietarinen & Snellman, 2006). It ties in with Peirce's game-theoretic conception of semantics and his notion of habits as stable, self-controlled tendencies.

First, understanding the fundamental nature of deductive reasoning in terms of EGs seems to be lurking in the iconic structure of graphs. We would thus be well advised to ask whether the graphs may be an aid in drawing deductive inferences over and above those accomplished by symbolic rules of inference? Peirce believed that reasoning is iconic, and in making inferences we are experimenting with diagrammatic representations. But even the simplest deductive inferences may involve creative considerations of *where* and *what* new individuals to add into the course of the proof, as for instance applying existential instantiation in countermodel constructions aptly demonstrate. We should expect deductive inferences to be facilitated when presented in the diagrammatic form of transformation rules. That this may indeed be the case is illustrated by some optimisation problems in automated theorem proving in which diagrammatic forms prove to be beneficial, though they are unlikely to solve the fundamental limits of what can be accomplished by mechanical traits of reasoning. According to Peirce, satisfactory deductive inference, let alone ampliative modes of reasoning, cannot at the end be accomplished by anything else than a "living intelligence" (MS 499).

Second, Peirce sought for the simple, "indecomposable elements of thought" that could constitute the primary building blocks of the complexes of our cognitive systems (MS 284:43 [1905], "The Basis of Pragmaticism"; MS 325:3 [n.d.], "Pragmatism Made Easy"). I have argued that, logically, indecomposable elements are the atomic graphs, "spots" in Peirce's terminology (Pietarinen, 2005a). The spots are nevertheless not diagrams but *images*, firstnesses of iconic signs that live on the *phaneron* (Pietarinen, 2010b). The interpretation of images is, unlike the interpretation of diagrams, singular and physiognomic. But Peirce emphasizes that the result need not be a simple quality (MS 280:17 [1905], "The Basis of Pragmaticism"). We can take these remarks to mean that indecomposable elements, as represented by the spots in EGs, are the iconic counterpart to what the interpretation of non-logical constants of the logical alphabet is in the symbolic realm. Spots, as specific bounded regions of the space of assertion and having some specific qualities by which they are distinguished from the surrounding space, are thus iconic just as logical constants are, but not in terms of being involved with observations of diagrammatic structures but in terms of being involved with qualitative imagery. Their intended interpretations are made possible precisely by virtue of them being such images. These interpretations provide the boundary conditions according to which the semantics for graphs may then be built. The intended interpretations may change following the changes in spots defined in terms of spatial and metric and not merely in terms of topological regions.

It would nevertheless be an error to take the theories of EGs to ally with the class of theories variously termed as mental models (Lakoff & Turner, 1989; Johnson Laird, 2002), cognitive spaces (Gärdenfors, 2000) or image schemas (Hampe, 2005), for example. Such cognitive theories take various spatial arrangements of conceptualizations to be the meanings of our expressions or assertions. According to the iconic language of logical diagrams, however, meaning is not confined to representations, because diagrams are signs, that is, representations that make themselves interpretable. Understanding complex expressions of diagrams requires semantic and pragmatic interpretation. To take meanings of complex assertions to somehow be located in those representations would imply a nominalist and internalist account according to which meaning *is* conceptualization in schemas, image-like qualities or mental models. But then there is nothing distinguishing such images being right or wrong about something or being true or false in a model.

Instead, EGs are externalist in that strong pragmatist sense of meanings as extra-linguistic, general habits of actions. There thus is a world of difference between, on the one hand, the presently popularized cognitive theories of semantics and cognitive semiotics, and on the other, the semantic/pragmatic theories of meaning planted by Peirce and developed further by Paul Grice and a few others (Pietarinen, 2004).

Consequently, the so-called 'Language of Thought' hypothesis is ready to be taken off the board. A postulation of internal, symbolic language beneath the logical level is from the point of view of the theory of EGs implausible: a brain-wired internal code cannot be relied on to determine which of the multiple readings of icons would be the intended ones. From the point of view of Peirce's theory of signs, it does not even make sense to pose the possibility of a symbolic level beneath an iconic one, because symbols are bound to involve indexical signs, and indexical signs are bound to involve iconic signs. That is to say that icons – images, diagrams and metaphors alike – are the closest we get at in terms of a rigorous logical representation of our cognitive thought operations. To claim otherwise is really to subscribe to the wide separation of cognition and meaning, which indeed had radicalized 20th century thought not only in the realm of symbolic logic in terms of its formal purification but also in those structuralist and formalist traditions in the studies of arts, culture and semiotics that attempted to create ideological barriers between language, thought and the world instead of attempting to unite them.[8] To put logic and cognition together again is to forever close the door to those lost paths.

> **Challenge B:** *Put logic and cognition together again by tackling the question of* EGs *as the dynamic, logical representation of intellectual cognition from the perspective of contemporary cognitive sciences.*

4. The disparity between logical diagrams and symbolic logic

Moving on towards the specifics of the logic of the beta part of the EGs, there are a couple of issues that have not been pointed out before. What the corresponding fragment of first-order logic is hinges a great deal on the details of how the theory and language of beta graphs are actually set up. For example, the usual presentations, including most of Peirce's own writings on the matter, assume all relation terms (graphically "the spots") to be symmetric. Peirce is aware of the need of adding a special proviso to be able to speak about all relations, including asymmetric ones. The 1902 dictionary entry observes that, "in taking account of relations, it is necessary to distinguish between the different sides of the letters" (Peirce, 1902, p. 649). When we do linguistic analysis, the lines connected to spots in beta graphs are normally to be read not only from outside in but also from left to right just as natural language is read. By 1905 Peirce acknowledges that relations would be widely conceived as soon as we give "relative significations to spots", so that "if a spot signifies an asymmetric relation it is necessary to distinguish connection with one part of it as meaning something different from connection with another side", adding that "colors or other qualities of lines" could be recognized to build up "a corresponding variety of asymmetric relations" (MS 284:90).

In the alpha part, there is no need for the operation of commutation, because we need not recognize "any order of arrangement [of propositional terms] as significant" (Peirce, 1902, p. 645). However, in beta graphs with specific spots and lines denoting asymmetric relations the sheets of assertion upon which these graphs are scribed must have orientation. Therefore we will lose the property of isotopy-equivalence according to which graphs can be observed from any angle in a meaning-preserving way.

Second, in beta there are no free variables. They could be introduced by fiat as certain selectives, but it is more recommendable to have a way of

[8] Greimas and Courtés (1982) should function as a warning sign. It is not an occupational hazard, for example, that their entry on "Semiotics" has no reference to Peirce at all.

doing so that is as iconic as possible. I suggest that free variables are taken as dots attached to the hooks within the interiors of the spots. They are not the dots or lines attached to the hooks at the peripheries of the spots that lie outside the boundaries of the spots. These are bound variables, according to Peirce's way of setting up the system of beta graphs: an attachment to the hooks outside the boundary refers to predication, and free variables do not predicate anything. When variables become bound, they will be extended from the hooks inside the spots to the corresponding hooks outside of the boundary.

Third, the theory of beta graphs does not distinguish well between proper names and singular terms, which are both treated by Peirce as predicate terms (spots) having some specific quality in terms of being regions of space in the sheet of assertion. This necessitates a complication in the transformation rules, as we do not want to infer from, say, "Barack Obama is a man" that "It is not the case that something is a man", in other words that "Everything is not a man". We come across such illicit inference if we are allowed to substitute the free end of the identity line within a negative area for a proper name that attaches to a singular term, as we can then apply the standard erasure and deiteration rules to the line which at once would permit the inference. A natural solution is to keep apart the notions of names ("selectives") and singular terms ("spots") and never substitute spots for names.

Fourth, in beta graphs the notion of scope is not a separate notion at all. The 'binding' scope is denoted by the directionality of identity lines spanning from outside-in and connecting different areas and spots. On the other hand, the nesting of areas corresponds to the 'priority' scope of quantificational constants. Unlike in first-order logic that makes heavy use of parentheses, these two notions do not go hand in hand in the iconic formation of logical constants. One might go as far as to say that there is no need for the primitive notion of scope in many-dimensional logical diagrams in the first place.

One particular consequence is worth mentioning here: the 'syntax' of iconic forms alone cannot tell us whether a dynamic or non-dynamic interpretation of quantification and its binding scope is intended. Consequently, beta graphs that dispense with the parenthetical notation can make use of such kinds of binding scopes that can reach beyond priority scopes, similarly as what can happen in dynamic extensions of first-order logic but what cannot be achieved in traditional first-order logic with more stringent scope conventions.

Challenge C: *Redefine the beta part so as to correspond not only to fragments of first-order logic but to full first-order logic and beyond.*

5. The gamma and the delta

Within the gamma realm we encounter a host of important issues which I will mostly forego here. Peirce occasionally referred to the planned delta part, which one needs "to add in order to deal with modals" (MS 500:3 [December 1911], "A Diagrammatic Syntax"). What was the delta part intended to be? Peirce had several systems of modal logics already in place, including quantification and multi-modal logics. But they were all introduced intermittently, and he was not able to expose their fundamental nature. He probably envisioned a unifying graphical account for all modality types, one that would encompass the tinctures, identity lines (quantification), and potentials, together with a feasible interpretation that would agree with his tenet of scholastic realism – which in contemporary terms is for all practical and logical purposes a suitably understood possible-worlds semantics (Pietarinen, 2006b). Presumably it was that unificatory challenge which was to be relegated to the delta part. However, we need to keep in mind that, even if all the modal notions were to be cut off from the gamma part, it would still leave that part to deal with graphs whose logical behavior is very different from one another, including higher-order logics, logic of collections, imperatives, erotetic logic, and even metaphors (Pietarinen, 2010c).

Challenge D: *Sort out the various gamma parts and recover the hidden delta.*

6. Non-classical and deviant EGs

The preceding issues deal with the background and general significance of the diagrammatic logic of EGs. Might we view Peirce's attempt as an early logic of our cognitive processes? Can it teach us something about the notion of information and information processing? Do the systems yield new perspectives to the meaning of logical constants? How to expand the method of representing logic using icons? Finally, let us summarize a couple of key logical matters pertinent to these questions.

The issue that naturally arises with regard to recent logical developments concerns the relationship between non-classical and deviant logics as well as the possible extensions and variations of the standard systems of EGs. Here is an abridged list of such lines of developments:

1. Intuitionistic versions take the *cut*, which is the icon of negation, to be an *incision* and not a *reversal* (Pietarinen, 2006a, p. 169). Thus a doubly cut proposition does not yield the proposition itself. Zalamea (2008) offers an alternative proposal as to how to get at an intuitionistic version of EGs by changing the iconic representation of the conditional. Which way to do it?

2. New modal systems for the gamma part can be developed by systematic variation of transformation rules. Some of them were studied long ago in Zeman (1964) but never taken much further. We need to place generic constraints on the transformation rules in order to generate different systems, and to study the relationship of such transformation rules to the accessibility relation in modal logic (Pietarinen, 2006b).

3. Peirce proposed representing higher-order notions, such as the relations of *anteriority* and *succession*, and which still are routinely considered to be Frege's sole discoveries, by a modification of gamma graphs to have spots as "potentials" that use abstraction and lines of identities as "objective possibilities" (MS 508, "Syllabus B.6"). Suggesting then some transformation rules for such higher-order graphs, he notes that they appear to result in incomplete systems of rules (ibid.) – as we know now second-order logic is indeed semantically incomplete. Since the semantics can be modified to weaker versions for semantically incomplete logics (Krynicki & Mostowski, 1995), the search for useful proof systems for higher-order languages need not be a dead end, however.

4. We ought to inquire about strict impossibility proofs as well. Is there something that cannot be represented by an iconic logic of EGs but is indispensable in symbolic languages? One candidate is the use of fixed points in logics, such as modal μ-calculus – it is not at all obvious what would be the essentially iconic component in recursion and fix-point operators. Another realm difficult to diagrammatize is provided by the multiplicative connectives familiar from linear logics. On the other hand, it is worth keeping in mind that these are both paradigm examples of such systems that may be born when the formal assails the semantic.

Challenge E: *Examine the issues 1–4 and assess their relevance with respect to Challenges A–D.*

7. Conclusion

Existential Graphs have a good claim to be the logic of our cognitive workings of reasoning and representation, along the lines of providing "a moving picture of the action of the mind in thought" (MS 298:1 [1905], "Phaneroscopy") as well as a "system for diagrammatizing intellectual cognition" (MS 292:41 [1906], Draft of "Prolegomena"). Their untapped logical potential is at the same time representative of the capacity of EGs becoming, as Peirce firmly believed, "the logic of the future". That potential has only been begun to be touched upon, and to fully argue for my bid calls for a continuing study of a combination of a number of logical and cognitive issues. Some of them have been raised here, such as the role of icons and images in logical theories, the meaning of logical constants and their cognitive economy, the reasons for the failure of the Frege–Russell thesis, and the reasons for the insufficiency of the mental model types of theories.[9]

References

Barwise, J. and Etchemendy, J. (1995). "Heterogeneous Logic." In B. Chandrasekaran, J. Glasgow, and N. H. Narayanan (Eds.). *Diagrammatic Reasoning: Cognitive and Computational Perspectives* (pp. , 209–232). Cambridge, Mass.: MIT Press.

Enderton, H. B. (1970). "Finite Partially Ordered Quantifiers." *Zeitschrift für Mathematische Logic und Grundlagen der Mathematik*, 16: 393–397.

Greimas, A. J. and Courtés, J. (1982). *Semiotics and Language: An Analytic Dictionary.* Bloomington: Indiana University Press.

Gärdenfors, Peter (2000). *Conceptual Spaces: The Geometry of Thought*. Cambridge, Mass.: MIT Press.

Hammer, Eric M. (2002). "Diagrammatic logic." In D. Gabbay and F. Guenthner (Eds.), *Handbook of Philosophical Logic 4* (2nd edition) (pp. 395–422). Dordrecht: Kluwer.

Hampe, B. (Ed.) (2005). *From Perception to Meaning: Image Schemas in Cognitive Linguistics.* Berlin: Mouton de Gruyter.

Hintikka, J. (1979). "Is, Semantical Games, and Semantical Relativity." *Journal of Philosophical Logic*, 8, 433–468.

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Johnson Laird, P. (2002). "Peirce, Logic Diagrams, and the Elementary Operations of Reasoning." *Thinking and Reasoning*, 8, 69–95.

Lakoff, G. & Turner, M. (1989). *More than Cool Reason: A Field Guide to Poetic Metaphor*. Chicago: University of Chicago Press.

Liu, X. (2008). "The Literature of C. S. Peirce's Existential Graphs." Institute of Philosophy, Beijing: Chinese Academy of Social Sciences. `http://philosophy.cass.cn/facu/liuxinwen/01.htm`

Krynicki, M. & Mostowski, M. (1995). "Henkin Quantifiers." In M. Krynicki, M. Mostowski and L. W. Szczerba (Eds.), *Quantifiers: Logics, Models and Computation 1* (pp. 193–262). Dordrecht: Kluwer.

Peirce, C. S. (1902). "Symbolic Logic or Algebra of Logic." In J. M. Baldwin (Ed.), *Dictionary of Philosophy and Psychology, Vol. 2* (pp. 640–651) New York and London: Henry Holt.

Peirce, C. S. (1906). "Prolegomena to an Apology for Pragmaticism." *The Monist*, 16, 492–547.

Pietarinen, A.-V. (2004). "Grice in the Wake of Peirce." *Pragmatics & Cognition*, 12, 295–315.

Pietarinen, A.-V. (2005a). "Compositionality, Relevance and Peirce's Logic of Existential Graphs." *Axiomathes*, 15, 513–540.

Pietarinen, A.-V. (2005b). "Cultivating Habits of Reason: Peirce and the Logica Utens versus Logica Docens Distinction." *History of Philosophy Quarterly*, 22, 357–372.

Pietarinen, A.-V. (2006a). *Signs of Logic: Peircean Themes on the Philosophy of Language, Games, and Communication*, Synthese Library 329. Dordrecht: Springer.

Pietarinen, A.-V. (2006b). "Peirce's Contributions to Possible-worlds Semantics." *Studia Logica*, 82, 345–369.

Pietarinen, A.-V. (2009). "Significs and the Origins of Analytic Philosophy." *Journal of the History of Ideas*, 70(3), 467–490.

Pietarinen, A.-V. (2010a). "Moving Pictures of Thought II: Graphs, Games, and Pragmaticism's Proof." *Semiotica*.

Pietarinen, A.-V. (2010b). "Peirce and the Logic of Image." *Semiotica*.

Pietarinen, A.-V. (2010c). "Iconic Logic for Metaphors." *Journal of Cognitive Science*.

Pietarinen, A.-V. and Snellman, L. (2006). "On Peirce's Late Proof of Pragmaticism." In T. Aho & A.-V. Pietarinen (Eds.), *Truth and Games* (pp. 275–288), Acta Philosophica Fennica 79. Helsinki: Societas Philosophica Fennica.

Roberts, Don D. (1973). *The Existential Graphs of Charles S. Peirce*. The Hague: Mouton.

Shin, S.-J. (2002). *The Iconic Logic of Peirce's Graphs*. Cambridge, Mass.: MIT Press.

Shin, S.-J. (2004). "Heterogeneous Reasoning and Its Logic." *Bulletin of Symbolic Logic*, 10, 86–106.

Sowa, J. (2006). "Peirce's Contributions to the 21st Century." In H. Schärfe, P. Hitzler, and P. Øhrstrøm (Eds.), ICCS 2006, *Lecture Notes in Artificial Intelligence 4068* (pp. 54–69). Berlin and Heidelberg: Springer-Verlag.

Wittgenstein, L. (1921). *Tractatus Logico-Philosophicus*. London: Kegan Paul.

Zalamea, F. (2008). "Intuitionistic Graphs." Manuscript.

Zeman, J. J. (1964). *The Graphical Logic of C. S. Peirce*. Dissertation, University of Chicago. Online edition, 2002, at `http://web.clas.ufl.edu/users/jzeman/`